Fertility Measures and Mathematical Modeling Predicts of Population Trends in Germany

Sridhar Akiri[*1] *Sri Sai Swetha*[2], *Aravind Battu*[3]

[1 2 3]*Department of Mathematics, GITAM School of Science*
GITAM (Deemed to be University), Visakhapatnam, Andhra Pradesh, India.

Abstract: *This study examines fertility and population estimation measures. The process through which the age distribution of a population adapts to alterations in living conditions is known as "demographic transformation." Consequently, socioeconomic developments result in a significant shift in the age distribution of a society. Fertility is one of the primary variables determining the natural increase of a specific geographical location; we quantify it as "Fertility Measures." The study of fertility rates tries to estimate the magnitude of future population growth so that requirements can be met accordingly. The population estimates of a country are essential for selecting the proper socio-economic and demographic development strategies. Consequently, population estimates lead to population projections for the region under investigation. The most recent dataset of Germany for the year 2021 is evaluated with all necessary parameters to compute basic fertility measures, and population projections are based on the total population of each country. Accordingly, inferences are made that fertility rates in Germany will increase from 1.53 to 1.57 by 2021. Population projections for the year 2025 are calculated using the population growth models.*

Keywords**:** Fertility measures, Growth Models, Population Estimates, Population Projections.

1. Introduction

Donald J. Bogue, 1969 [1] defined demography as the mathematical and statistical study of the size, composition, and spatial distribution of the human population and of changes over time in these aspects through the operations of the five processes of fertility, mortality, marriage, migration, and social mobility. Connor, H. D., 2022[3], in his paper, described a statistician from England named "John Graunt" as the “Founding father of human demography, epidemiology, and vital statistics,” and he is credited with developing the field of demography through his statistical analysis of human populations. Graunt was the first to rationally estimate the population's size and establish the ratio of boys to girls as the proportion of births. Demographic data collection and usage have been practiced worldwide since the Ancient Greeks. Demographic information, which is frequently obtained through a census, has been recorded on ancient scrolls, tombstones, and tombstones, and in works of literature from the time. A population is described and defined by its demographics. Primary demographic data is gathered on subjects including sex, gender, age, education, culture, religion, earnings, civil status, family structure, handicap status, and house ownership, among others. Beyond governmental planning and counting, demographics help select the intended audience and populations for campaigns, raising donations, marketing, and advertising, and determining the attributes of possible research subjects.

Data collection on live births, deaths, migration, fetal deaths, marriages, and divorces is known as vital statistics. Civil registration, an administrative system in use by governments to record important events that happen in their communities, is the most popular method of gathering information on these occurrences. While keeping track of important occurrences for legal purposes, these records are also a crucial source of statistical data. To detect and quantify health-related concerns as well as track advancement towards improving quality and public health objectives, data from vital records is essential. The formulation of policies, the funding of projects and research, and evaluations of the quality of service are all influenced by crucial indicators of the country's health, particularly maternal and new-born mortality. One of the critical determinants of demography is fertility. The ability to conceive a child naturally is known as fertility. Fertility is a factor in population increase, revealing both the effects and causes of societal and economic changes, along with death and migration. Different intermediary variables have varying effects on reproduction, depending on the society. Age, weight, exercise, education, lifestyle, marriage, mental well-being, drugs, and many other factors can all impact fertility. D.T. Baird and Team, 2010 [4], Catherine Hakim, 2003[5], H. Kulu, 2005 [6] discuss the factors affecting fertility i.e. family life & Migration in their respective papers. Monitoring fertility rates enable more effective and good planning and resource distribution within a specific region.

One of the main factors affecting the population growth rate is the total fertility rate (TFR). While predicting the changes in population size, demographers mainly focus on key determinants, namely fertility & mortality rates, migration, and age profile of the population. A population projection depicts how the population may look in terms of size, age, and sex composition in the future. Population forecasts and estimations influence the actions of government institutions and businesspeople. It aids the development of appropriate solutions by assisting policymakers in understanding the current issues. There are three methods to project the population; the one we used here for analysis is the mathematical model, then comes the economic method and the cohort-component method. The mathematical model includes Arithmetic, Geometric, Exponential, Gompertz, logistic, Makhem, Modified exponential, hyperbolic, regression, and polynomial growth models. Coming to the geographical region of study: Germany, a Western European country with a population share of 8.3% in the continent, stands second among all the other European countries in terms of population. Germany had a sharp decrease in birth rates in former East Germany after reunification in 1990 and in the early 1930s following the Great Depression. The birth rate in that region of the country reached a record low of 1.2 children per woman as a result of the ensuing economic downturn.

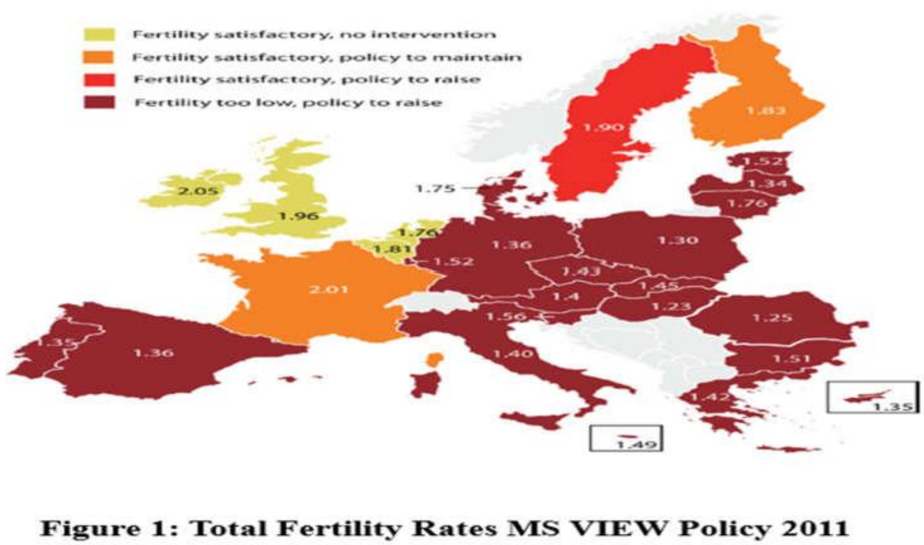

Figure 1: Total Fertility Rates MS VIEW Policy 2011

2. Literature Survey

Bos, E., Vu, M. T., Massiah, A., Bulatao, R. A. ,1994 [2] highlights trends in demographic indicators in different regions of the world and describes the methodology used in making the projections, including procedures for incorporating AIDS mortality. Ramesh Adhikari, 2010 [7] examined the demographic, socioeconomic, and cultural factors for fertility differentials in Nepal. This paper has used data from the Nepal Demographic and Health Survey (NDHS 2006). Alicia Adsera,2017 [8] discusses the fact that the evolution of fertility totally depends on women's education and their decision to childbearing. The author mainly focused on the actual fertility rates of the highly educated and whether they will have more children in the future. This article by Gorge C. Alter,2019 [9] examines the development of historical demography from the 1950s to the present by examining the development of its methods. It focuses on a few influential methodological developments but omits many important substantive contributions. Azose, J. J., Ševčíková, H., & Raftery, A. E.,2016 [10] studies the uncertainty in migration projection which is a substantial contributor to uncertainty in population projections for many countries.

Eckart Bomsdorf, Bernhard Babel & Jens Kahlenberg, 2010 [11] study the problem of an aging society and tells that a standard measure for quantifying this effect is the old-age dependency ratio (ODR). John Bongaarts and Tomas Sobotka,2012 [12] studied the period and cohort fertility indicators in the Czech Republic, the Netherlands, Spain, and Sweden and showed that the newly adjusted measure has an excellent fit with the completed fertility of women in their peak reproductive years. Kamila Cygan-Rehm, 2014 [13] discussed the main argument that immigrants' fertility is influenced by the fertility culture of their native nation. Women from nations with high fertility rates give birth to more children than women from nations with low fertility rates do. A significant amount of the fertility differences between immigrants and German residents can be attributed to home country TFRs. This paper by Diprete, T. A., Morgan, S. P., Engelhardt, H., & Pacalova, H, 2003 [14] discusses the Parity-specific probabilities of having a next birth estimated from national fertility data. Nation-specific costs of having children are measured by time-budget data, attitude data, and panel data. Empirical analysis supports the assertion that institutionally driven child costs affect fertility patterns.

Y L. L. Eberhardt w,1987 [15] discusses the simple differential equation models for populations subjected to extensive removals are used to estimate potential rates of growth in the presence of removals, using the ratio method and a minimum chi-square fitting procedure. England. K and Natasha Azzopardi-Muscat, 2017 [16] focus on Europe's aging population and declining fertility rates. The public health of Europeans is strongly influenced by unemployment, shifts in the age composition of the population, and other demographic changes. Therefore, public health should highlight the idea of healthy aging. The authors F Billari &HP Kohler, 2004 [17] depicted the relationship between low and lowest-low periodic fertility and cohort fertility. Also, they resulted in critical fertility-related behaviour of the human population. The cross-country difference in fertility patterns has also been discussed based on the analysis results. Arthur J. Knodel, 2015 [18] gives us a detailed statistical description and analysis of birth rates in Germany between unification and World war- II. The publisher used simple and modern statistical methods to compute the demographic trends in Germany and to explain the fall in fertility that occurred in the late 19th century.

Marek Kupiszewski, Jakub Bijak, and Beara Nowak, 2008 [19] examine the influence of future changing demographics on multiple socioeconomic domains, such as education, labour market, health, and so on, and recommend feasible policies for future trends. It also examines population trends over the next 45 years, using population data from 2005. Wolfgang Lutz, 2006 [20] talks about two things: Summarize arguments that either suggest the assumption of a recovery of fertility rates in Europe or imply further declines. Anna Matysiak, 2020 [21] explains that during and before the great recession in Europe from 2002 to 2014, fertility was impacted. Findings indicate a strong correlation between the fall in fertility and the rise in unemployment. M Kreyenfeld and D Konietzka, 2008 [22] discussed the fertility decline in West Germany due to education and Gender Equity in their publication. The overview is that the childbearing age difference between highly-educated women and low-educated women has increased tremendously after unification which contributed to great educational differences in the fertility behaviour of the unification cohorts.

Xizhe Peng, 2011 [23] concentrates on the topic of China is at a demographic turning point: It is changing from an agricultural society into an urban one, from a young society to an old one, and from a society attached to the land to one that is very much on the move. Olga Potzsch, 2016 [24] tells us about the population projections in Germany issued by Federal Statistics Office as the 2011 census has shown a decrease in fertility rates in the following decade, i.e., 2020's. This mainly focussed on the analysis of cohort fertility and the consequences of late entry into motherhood on the completed fertility. Tomas Sobotka, 2008 [25] tends to focus on immigrant women, who have significantly greater levels of menstrual fertility than 'indigenous' populations. Still, this difference diminishes with time and longer stays in the country. Immigrants contribute significantly to overall babies born, and their share has risen over the last decade. Stoto, M. A., 1983 [26], studies past population projection errors, which provide a means for constructing confidence intervals for future projections. Finally, she demonstrates that total population size projections and simple projection techniques are more accurate than more complex techniques.

Simon Szreter, 1993 [27], mainly focused on the intellectual history of the idea of demographic transition: the idea that has provided students of changing fertility throughout the post-war era with the dominant collective definition of the phenomenon they seek to understand and explain. This paper, described by James C Witte & Gert G Wagner, 1995 [28] talks about declining fertility rates in Eastern Germany than that of West Germany, which already had the lowest fertility in the world in 1990 due to unification. The writers used a broad sociological perspective to view demographical trends as well as attitudinal and behavioural changes which come with unification.

3. Methodology:

Notations:

P_0– the initial size of the population
P_t – population at the time 't.'
r_A – Intrinsic growth rate for arithmetic model
r_G – Intrinsic growth rate for geometric model
r_E – Intrinsic growth rate for exponential model

3.1 Measures of fertility

3.1.1 Crude birth rate (CBR): It is described as a straightforward proportion between the total number of live births and the total population of a given year. Symbolically it is given as:

$$CBR = \frac{Total\ residekt\ live\ births}{Total\ populatiok} \text{ X } 1000$$

In general, the CBR is calculated as per 1000 of the population, and the denominator needs to be an average population size of that particular year.

3.1.2 General Fertility Rate (GFR): It may be described as the proportion between the total number of births in a given year and the total number of women who are 15 to 49 years old and capable of bearing children.

$$GFR = \frac{Births\ in\ a\ stated\ period}{Mean\ no.\text{of women } aged\ 15-49\ in\ the\ same\ period} \text{ X } 1000$$

It is also calculated per 1000 population of the childbearing women population.

3.1.3 Age-Specific Fertility Rate (ASFR): The age-specific fertility rate is used to depict the annual births of women of specific age groups. Mathematically ASFR can be represented as the ratio of births to women of age group ‘x’ to ‘x+n’ in the specific period to the overall gender breakdown for the age group ‘x’ to ‘x+n’ in that specific period of time calculated per 1000 population. Symbolically it is given as:

$$ASFR = \frac{No.\text{ of } births\ to\ womek\ ik\ the\ age\ group'x'-'x+k'}{No.\text{ of Wom}ek\ aged'x'-'x+k'} \text{ X } 1000$$

The term ‘x’ to ‘x+n’ generally takes the age interval gap of 5 years between the childbearing age 15-49 years.

3.1.4 Total fertility rate (TFR): The total fertility rate refers to the total number of births a woman can give birth to during her childbearing years. Mathematically it is the product of the sum of the age-specific fertility rate of the childbearing ages, i.e., 15-49 years, to the intervals of age groups, which is generally taken as five years.

Symbolically it can be represented as:

$$\text{TFR} = \frac{\sum_{x=15-19}^{45-49} \text{ASFR}}{1000} \text{X } 5$$

3.1.5 Gross Reproductive Rate (GRR): The Gross Reproductive Rate is regarded as the extent to which the generation of daughters can replace the present generation of females. Mathematically it can be defined as the number of female births to the total number of females in the period childbearing age group. It is also computed per 1000 population. Symbolically it can be represented as:

$$GRR = \frac{No.\text{ of } Female\ births}{Total\ No.\text{ of } births} \text{ X} TFR$$

3.1.6 Net Reproductive Rate (NRR): The capacity to reproduce changes from generation to generation as a result of both natural and manmade influences, which also affects the Number of women in the population. The Net Reproductive Rate is used to calculate the change in fertility. This can be described as the ratio of the Number of female children surviving per 1000 population. Symbolically it can be represented as:

$$NRR = \frac{BIRTHS-DE\text{Æ}THS}{TOT\text{Æ}L\ POPUL\text{Æ}TION} X100$$

3.2 Growth Models:

3.2.1 Arithmetic Growth Model: This approach entails calculating the typical rate of population variation over time. Here, we use the assumption that the population is increasing steadily over time and that it exhibits the straight-line trend shown by

$$P_t = P_0(1 + r_A t)$$

Where,

$$r_A = \frac{1}{t}\left[\frac{Pt}{P0} - 1\right]$$

This assumption doesn't hold well for a short period and when the amount of change is small.

3.2.2 Geometric Growth Model: According to the assumption of the geometric growth model, population size multiplies itself in a short period of time and is given by,

$$P_t = P_0 (1 + r_G)^t$$

Where,

$$r_G = aktilog\left[\frac{1}{t}\log_e \frac{Pt}{P0}\right] - 1$$

This model implies huge growth of population.

3.2.3 Exponential Growth Model: If the growth of the population is compounded geometrically, then in the long-range the population grows exponentially, given by

$$P_t = P_0\ r_E\ t$$

Where,

$$r_E = \left[\frac{1}{t}\ln \frac{Pt}{P0}\right]$$

If the population grows exponentially, then it is predicted that in the near future population explosion will take place.

Age Group	Number of Births	Total Births	Population	Total population	Deaths

4. Implementation:

The dataset i.e. Table 1 used for implementation is taken from the official census website of Germany (https://www-genesis.destatis.de/) and the dataset is sorted based on our requirements.

Total Population(2021) - 8,32,37,124

Total Population (2011) – 80274983

Table 1. Dataset for Implementation

	Male	Female		Male	Female		
15-19 years	4367	3999	8366	1975398	1851582	3826980	894
20-24 years	30695	29241	59936	2364238	2158289	4522527	1358
25-29 years	91412	87105	178517	2545675	2347054	4892729	1682
30-34 years	159226	150691	309917	2865263	2688341	5553604	2670
35-39 years	97323	92445	189768	2717782	2617412	5335194	4142
40-44 years	23982	22206	46188	2580030	2552269	5132299	6175
45-49 years	1284	1187	2471	2435702	2428702	4864404	9920
TOTAL	408289	386874	795163	17484088	16643649	34127737	26841

4.1 Calculation of Measures of Fertility:

The following tables give the values obtained on the implementation of the methodology

Table 2.1 gives the values of fertility measures

Table 2.1. Values Obtained on Implementation of the Methodology

Measures of Fertility	Obtained values
CBR	9.5529
GFR	47.7757
ASFR	315.2468
TFR	1.5762
GRR	0.7692
NRR	0.9231

Table 2.2 gives the population estimates.

Table 2.2. Values obtained on implementation of the methodology

Growth Model	Intrinsic Growth rate (r)	Projected Population
Arithmetic Growth model	0.0036	8,62,33,660
Geometric Growth Model	0.0036	8,62,82,673
Exponential Growth Model	0.0036	8,62,88,251

The graph below (Figure 2) gives the change in the population of Germany in the years 2011,2021 &expected population estimate of 2025 & the graph is plotted using Excel.

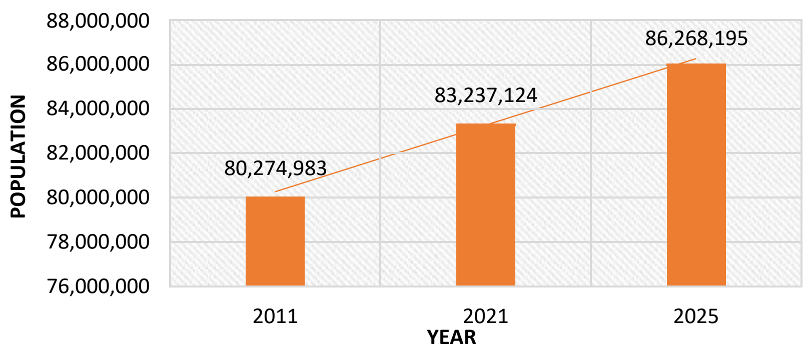

Figure 2 . Graph of 2011,2021,2025 Populations of Germany

5. Conclusion:

The important life events of births, deaths, and migration are addressed the most in demography. Population growth is primarily caused by fertility, which is quantifiable through fertility measures. These values were attained: 9.5529 is the crude birth rate, 47.7757 is the general fertility rate, 1.5162 is the total fertility rate, 0.7692 is the gross reproduction rate, and 0.9231 is the net reproduction rate. In comparison to fertility rates in 2020, their respective amounts have increased. The fertility rates in Germany grew by 0.38% from the fertility rates in 2020. One of the simplest ways to anticipate the future population that uses the present demographics and circumstances is to use mathematical models for population projections. The effective use of population projection models is then used to anticipate the population of Germany in 2025. Populations impacted by applying geometrical, mathematical, and exponential models in Germany's projected populations are 86233660, 86282673, and 862882511. The estimates indicate that Germany's population will rise by 1.4% between 2021 and 2022, with only an intrinsic growth rate of 0.0036.

In conclusion, these conclusions for Germany are based on the fundamental fertility measurements. If current patterns in female academic achievement and utilization of contraception continue, fertility rates will fall faster, and population increase will be slower, according to our study. A sustained TFR below the replacement level would have adverse financial, social, ecological, and geopolitical consequences in many countries, including Germany. It will be crucial in the future years to find policy options that will support continued low fertility yet maintain and enhance female reproductive health.

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