Cosmological models with time dependent cosmological and gravitational constants

Kangujam Priyokumar Singh

[1]Department of Mathematical Sciences, Bodoland University, India

In this paper we have investigated the problems on physical distributions involving the Newton's gravitational constant G and cosmological constant Λ which is considered varying as H^2 .The field equations and their solutions for perfect fluid in Robertson-Walker universe for all the values of scale factor are discussed and we find the exact solutions satisfying the law $\Lambda \infty t^2$.

Keywords: Cosmological constant, four velocity vector, cosmic time.

INTRODUCTION

The problem of cosmological constant is one of the most salient and unsettled problem in cosmology. To resolve the problem of huge difference between the effective cosmological constant observed today and vacuum energy density predicted by quantum field theory, several mechanisms have been proposed by (Weinberg, 1989). A possible way is to consider a varying cosmological term. From the reason of coupling of dynamic degree of freedom with the matter fields of the universe, Λ relaxes to present small value through the expansion of the universe and creation of the particles. From this point of view, the constant is small because the universe is old. The idea of G in the frame work of general relativity was first proposed by (Drac, 1937 and Lau, 1937) working in the frame work of general relativity proposed modification linking the variation of G with that of Λ. Keeping in mind, several authors have studied the problems on physical distributions involving the Newton's gravitational constant G and cosmological constantΛ, which are normally considered as fundamental constants varying in time as R^{-2}, where R is the scale factor of our expanding universe. This physical property has been investigated under some general assumptions in conformity with quantum cosmology by (Chen and Wu, 1990). It has not been further observed that the time varying Λ does not lead to any conflict with the existing observations. At the same time the cosmological constant Λ varying with time to time leads to the creation of matter with a rate at present comparable to that in the steady-state cosmology. Not only the above mentioned researchers a lot of work in this area were done by many other authors like (Carvalho and Lima, 1990; Schuzhold, 2002; Sivaram et al., 1975; Peebles and Ratra, 2003; Priyokumar and Ibotiombi, 2011 and Priyokumar, 2010).

The problem of physical distribution involving variable Gravitational and Cosmological constant has been further studies by the author (Berman, 1991). The author has considered the Einstein field equation with perfect fluid as source and variables Λ and G for the Robertson-Walker metric. The author also has investigated the problem by considering the perfect gas equation of state models in the Euclidean and non-Euclidean cause with assumption that the deceleration parameter is constant with the postulation of conserving of energy momentum. The results give the rise to explanation of the involvement of value of cosmological term in the early universe.

We have further investigated the problem investigated

by (Berman, 1991) with the assumption that
$\Lambda \propto H^2$.

The physical justification of the assumption is that (Berman, and Gomide, 1988) have already derived physically realistic solutions for flat universe giving the solutions.

$G\rho = At^{-2}$

and

$\Lambda = Bt^{-2}$

with an assumption involving G and H varying as t^{-2},

where $H = \frac{\dot{R}}{R} = \frac{1}{mt}$, $m \neq 0$.

In selection2 we have presented the field equations in the Robertson-Walker universe corresponding to all the value of curvature index K. The physical interpretations of the solutions obtained are discussed in section 3.

Field equation and their solutions

The metric considered for the present problem is the Robertson–Walker metric

$$ds^2 = dt^2 - R^2(t)\left\{\frac{dr^2}{1-Kr^2} + r^2 d\theta + r^2 \sin^2\theta d\phi^2\right\}, \quad (1)$$

where 't' is the cosmic time; R(t) , the scale factor of the universe; and K, the curvature index which takes up the value +1, 0 and -1.

The energy momentum tensor $T_{\mu\nu}$, for the perfect fluid is given by

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}, \quad (2)$$

where ρ is the matter density, p is the pressure and u^μ Is the four- velocity vector satisfying

$$u^\mu u_\mu = 1, \quad (3)$$

By use of the co-moving co-ordinate system, we obtain

$u^1 = u^2 = u^3 = 0$ and $u^4 = 1$, (4)

The Einstein field equation with time dependent cosmological and gravitational constants is

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}, \quad (5)$$

where $R_{\mu\upsilon}$ is the Ricci tensor, G and Λ being the time dependent variable gravitational and cosmological terms.

The usual conservation law is

$$T^{\mu\nu}{}_{;\nu} = 0, \quad (6)$$

The above equation (6) leads to

$$\rho \dot{G} = -\frac{\dot{\Lambda}}{8\pi}, \quad (7)$$

and $\dot{\rho} = -3\frac{\dot{R}}{R}(\rho + p)$, (8)

The field equation (5) for the Robertson-Walker metric and perfect fluid source reduce to the following pair of equations

$$\frac{3\ddot{R}}{R} = -4\pi G[3p + \rho - \frac{\Lambda}{4\pi G}], \quad (9)$$

and $\frac{3\dot{R}^2}{R^2} = 8\pi G(\rho + \frac{\Lambda}{8\pi G}) - \frac{3K}{R^2}$, (10)

We make use the equation of state

$p = \in \rho$, (11)

where $0 \leq \in \leq 1$.

Now we impose the assumption that

$\Lambda \propto H^2$ i.e. $\Lambda = 3\beta H^2$, (12)

where β is the dimensionless number of order of unity, with the factor 3 being introduced for our mathematical convenience as in Carvalho et. al., (1985)

By eliminating G and ρ from equations (9) and (10), we obtain

$R\ddot{R} + a\dot{R}^2$ = -bk, (13)

where a = (b+1) (1-β) - 1 and b = $\frac{1}{2}(3\in + 1)$

Case I

For a flat universe, i.e., K= 0, equation (13) becomes

$R\ddot{R} + a\dot{R}^2 = 0$, (14)

The general solution for the scale factor R obtained from the equation (14) is

$R = At^m$, (15)

where A = $\left\{(a+1)c'\right\}^{\frac{1}{a+1}}$ and m = $\frac{1}{a+1}(a \neq -1)$,

and c' is the constant of integration.

From equation (12) , we obtain

$\Lambda = Bt^{-2}$, (16)

where $B = 3\beta m^{-2}$.

The above equation (16) implies that

$\Lambda \propto t^{-2}$, (17)

From equation (10), we obtain

$G\rho = c_1 t^{-2}$, (18)

where $c_1 = \frac{3}{8\pi}(1-\beta)m^2$.

From equations (7) and (8), we obtain

$$G = Dt^{N}, \tag{19}$$

where D is a positive constant and $N = \dfrac{B}{4\pi c_1}$.

From equation (18), we obtain

$$\rho = \frac{c_1}{D}\, t^{-(2+N)}, \tag{20}$$

Using equation (8) , we can deduce

$$p = \frac{c_1}{D}\left\{\frac{(N+2)(a+1)}{3} - 1\right\} t^{-(2+N)}, \tag{21}$$

From equations (20) and (21), we obtain

$$\alpha = \frac{(N+2)(a+1)}{3} - 1, \tag{22}$$

Case II

For closed model of the universe (K =1), equation (13) becomes

$$R\ddot{R} + a\dot{R}^2 = \pm b, \tag{23}$$

The solutions of equation (23) is

$$R = Q\,t, \tag{24}$$

where $Q^2 = \pm \dfrac{b}{a}$

Using equation (24) in (12), we get

$$\Lambda = 3\beta t^{-2}, \tag{25}$$

From equation (10), we get

$$8\pi G\rho = 3\left(1-\beta+\frac{1}{Q^2}\right)t^{-2}, \tag{26}$$

From equation (7), we obtain

$$8\pi\rho G = 6\beta\, t^{-3}, \tag{27}$$

Dividing (27) by (26), we get

$$G = t^{2\beta Q^2 \left(Q^2 - \beta Q^2 + 1\right)^{-1}}, \tag{28}$$

From equation (27) , we obtain

$$\rho = E\,t^{n}, \tag{29}$$

where

$$E = \frac{3}{8\pi}\left(1-\beta+Q^{-2}\right)$$

and

$$n = \frac{-2\left(Q^2+1\right)}{Q^2 - Q^2\beta + 1}$$

Using equation (8) , we obtain

$$p = -\frac{1}{3}(n+3)\,E\,t^{n}, \tag{30}$$

which leads to

$$\alpha = -\frac{1}{3}(n+3), \tag{31}$$

Case III

Also for the open model of the universe, i.e., K = -1,

We have from equation (10)

$$8\pi G\rho = 3\left(1-\beta-Q^{-2}\right)t^{-2}, \tag{32}$$

From equation (7), we obtain

$$8\pi G\rho = 6\beta\, t^{-3}, \tag{33}$$

Using (32) and (33), we get

$$G = t^{2\beta Q^2 \left(Q^2 - \beta Q^2 - 1\right)^{-1}}, \tag{34}$$

From equation(33) , we get

$$\rho = E_1 t^{n_1}, \tag{35}$$

where $E_1 = \dfrac{3}{8\pi}\left(1-\beta-Q^{-2}\right)$.

and

$$n_1 = \frac{-2\left(Q^2-1\right)}{Q^2 - \beta Q^2 - 1}$$

Using equation (8), we obtain

$$p = \frac{1}{3}\left(n_1+3\right)E_1 t^{n_1}, \tag{36}$$

which leads to

$$\alpha = -\frac{1}{3}\left(n_1+3\right), \tag{37}$$

Physical Interpretation of the solutions

Berman and Gomide (1988) have already derived physically realistic solutions for flat Universe considering perfect fluid as energy momentum tensor with the assumption that the cosmological constant, $\Lambda \propto t^{-2}$.

In the present paper, we have further studied the problem with the assumption that $\Lambda \propto H^2$ corresponding to three values of curvature index K of the cosmological universe i.e. K= +1, 0 and -1.

Corresponding to the Case I where K= 0 for the flat universe, we have from equation (15) that the scale factor R of the Robertson-Walker metric, increases with time in the flat model of the universe subject to the condition that

$\frac{1}{a+1} > 0$ and m>0.

In this case the cosmological constant Λ→ 0 as the cosmic time t→ ∞ while the cosmological constant Λ is not defined at the initial epoch. The gravitational constant G is found to be a function of cosmic time'ť' provided the arbitrary constant B ≠ 0.

For a physically realistic solution of the matter density ρ we must have ρ≥ 0, which implies that $\frac{C_1}{D_1} \geq 0$, where C_1 and D are arbitrary constants.

From (20) we observed further that the matter density ρ will be an algebraic decreasing function of time when 2+N > 0, where N is an arbitrary constant. Similarly from (21) we observed that the pressure p will be decreasing function of time provided 2+N >0. At the same time the pressure p will be positive provided

$$\frac{(N+2)(a+1)}{3} - 1 > 0$$

Under, Case II we consider solutions for a closed universe, where the curvature index K = +1. We observe from equation (24) that the radius of the universe varies directly as the linear function of cosmic time'ť'.

For a realistic solution we should have the coefficient of time t, Q = + b⁄a.

We further obtain that the cosmological constant,

$$\Lambda \propto t^{-2}$$

From equation (28) we observe that the gravitational constant G varies directly as a linear function of time provided

$$\frac{2\beta}{1-\beta+Q^2} = 1$$

From equation (29) and (30), we observe that

$$p = -\frac{1}{3}(n+3)\rho$$

Then corresponding to the values of n = -3, -6, and -4
We obtain

p = 0, (dust distribution),
p = ρ, (stiff fluid distribution),
and $p = \frac{1}{3}\rho$, (disordered distribution of radiation) respectively.

It shows further that both density and pressure will decrease as time'ť' increases and both will tend to zero as $t \to \infty$.

Under Case III for the open model of the universe K= - 1, we have from equation (35) and (36) that

$$p = -\frac{1}{3}(n_1+3)\rho .$$

In this case also the problem reduces to the cases of dust distribution, stiff fluid distribution and disordered distribution of radiation. Corresponding to the values of n_1= -3, - 6 and – 4 respectively.

We can make further conclusion that both the matter density ρ and pressure p are found decreasing functions of time ultimately tending to zero as 't' tends to infinity.

In all the cases $n_1 < 0$,it implies that

$$\frac{2(Q^2-1)}{Q^2-\beta Q-1} > 0$$

i.e. $Q^2 - 1 > 0$

i.e. $Q > \pm 1$

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