

# Disease diagnosis using an advanced distance measure for Intuitionistic Fuzzy Sets

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## ABSTRACT

In the application of intuitionistic fuzzy sets (IFSs), distance measures play a very important role in decision science. Though different distance measures of IFSs are developed with various aspects, many of them do not satisfy the axioms of distance measure or encounter some counterintuitive cases. To fill up the deficiency it is always useful to define novel distance measures, which motivates us towards the development of new distance measure. This paper presents a new method for distance measure between IFSs. For the efficiency and validity of the proposed distance measure a comparative study is carried out with the numerical examples and also the distance measure is applied in a multi criteria decision making process. Finally, two medical diagnosis problems are discussed under this setting.

**Keywords:** Multi criteria decision making, distance measure, intuitionistic fuzzy sets, medical diagnosis.

## INTRODUCTION

In many of real world situations, usually decision makers are confronted with multiple criteria to be considered before any decision can be made. In Decision science, one method which has a great importance in the field of research is TOPSIS. Distance measure is one of the most integral parts of this method. It was developed by Hwang and Yoon (Hwang et al., 1981). Multi criteria decision making process provides the best alternative among a set of alternatives in the presence of different criteria of the alternatives. Crisp data are not always adequate to model in many real life situations where as fuzzy set theory is more suitable to handle such type of situation. Further, Atanassov (Atanassov, 1986) developed the theory of Intuitionistic fuzzy set (IFS), a generalised concept of fuzzy set theory (FST). In FST, to each element of the universe of discourse a degree of membership between 0 and 1 is assigned and the degree of non membership is considered as complement to the membership degree. On the other hand, IFS does not imply that the non membership degree is always the complement of the membership degree. Instead, it characterised some hesitation degree between membership and non membership

degrees. That is why, IFSs is more suitable to handle uncertainty than FST. Now a days, IFS theory becomes more popular for the uncertainty modelling problem and applied in a wide range of areas, such as, decision making, medical diagnosis (de et al., 2001, Szmit et al., 2005), fuzzy optimization, pattern recognition (Hung et al., 2004, Li et al., 2002, Li et al., 2007, Liang et al., 2003, Mitchell et al., 2003, Vlachos), logic programming (Atanassov et al., 1990), Distance measures, Divergence measures and similarity measures (Chen et al., 1994, Chen et al., 2016, Deng et al., 2015, Du et al., 2015, Beliakov et al., 2014, Zhang et al., 2014, Xu, 2017, Du et al., 2015, Hatzimichailidis et al., 2012, Ngan et al., 2018) are the important content in IFSs. Szmidt and Kacprzyk (Szmidt et al., 2000) proposed four distance measures of IFSs, which were based on the geometric interpretation of IFSs. Later, Grzegorzewski (Grzegorzewski et al., 2004), Szmidt and Kacprzyk (Szmidt et al., 2004) modified these distance measures. Further, Wang and Xin (Wang et al., 2005, Park et al., 2009) developed several distance and similarity measures of IFS (Yang et al., 2012). Singh and Garg (Singh et al., 2016) developed distance measures between the type-2 IFSs. Recently, Garg and

Rani (Rani et al., 2017), Garg and Kumar (Garge et al., 2018, Garge et al., 2018 Garg et al., 2016, Garge et al., 2017), Garg and Arora have proposed distance measures of IFSs (Garg et al., 2017).

This paper presents a novel distance measure of IFSs. To show the validity, efficiency and applicability of the proposed distance measure a comparative study with numerical examples and two medical diagnosis problems are carried out through a multi criteria decision making methodology. The detail work has been shortened as follows. Preliminaries starts with some relevant preliminary definitions. In propose distance measure between IFSs, a novel distance measure is proposed and numerical analysis has been done. Application of the proposed distance measure in Multi criteria decision making discusses the multi criteria decision making methodology by using the proposed distance measure. Two case studies of medical diagnosis have been studied through the methodology in Case study. Results and discussion are presented in Results and Discussion respectively.

Finally, a concrete conclusion has been drawn in conclusion.

**PRELIMINARIES**

In this section, some basic concepts and necessary backgrounds of FST, IFSs are reviewed (Zadeh, 1965, Dutta et al., 2018).

**Fuzzy Set**

Fuzzy set is a set in which every element has degree of membership of belonging in it. Mathematically, let X be a universal set. Then the fuzzy subset A of X is defined by its membership function ;  $\rightarrow [0, 1]$ , which assign a real number  $\mu$  in the interval  $[0, 1]$ , to each element  $x \in X$ , where the value of  $\mu$  at x shows the grade of membership of x in A. i.e.,  $A = \{ (x, \mu(x)) : \mu \in [0, 1] \}$ .

**Intuitionistic Fuzzy Set**

A Intuitionistic fuzzy set A on a universe of discourse X is of the form  $A = \{ (x, \mu(x), \nu(x)) : x \in X \}$ , Where  $\mu(x) \in [0, 1]$  is called the “degree of membership of x in A”,  $\nu(x) \in [0, 1]$  is called the “degree of non- membership of x in A”, and  $\mu(x)$  and  $\nu(x)$  satisfy the condition that  $0 \leq \mu(x) + \nu(x) \leq 1$ . The amount  $\pi(x) = 1 - \mu(x) - \nu(x)$  is called hesitancy of x

which is reflection of lack of commitment or uncertainty associated with the membership or non-membership or both in A.

**Metric**

Let X be a nonempty set and A, B and C are IFSs on X. Then the function is a metric (distance) if it satisfies the following axioms:

1.  $0 \leq d(A, B) \leq 1$
2.  $d(A, B) = 0 \iff A = B$
3.  $d(A, B) = d(B, A)$
4. if  $A \subseteq B \subseteq C$ , then  $d(A, C) \geq d(A, B)$  and  $d(A, C) \geq d(B, C)$

**Distance Measure Between IFSs**

Atanassov (Atanassov, 1999) suggested a direct generalization of the above distances for IFSs. For any two IFSs  $A = \{ (x, \mu_A(x), \nu_A(x)) : x \in X \}$

$$B = \{ (x, \mu_B(x), \nu_B(x)) : x \in X \}$$

The Hamming distance  $d_H(A, B)$ :

$$d_H(A, B) = \frac{1}{2} \sum_{x \in X} [|\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)|]$$

The normalised Hamming distance  $d_{NH}(A, B)$ :

$$d_{NH}(A, B) = \frac{1}{2} \sum_{x \in X} [|\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)|]$$

The Euclidean distance  $d_E(A, B)$ :

$$d_E(A, B) = \sqrt{\frac{1}{2} \sum_{x \in X} [(\mu_A(x) - \mu_B(x))^2 + (\nu_A(x) - \nu_B(x))^2]}$$

The normalised Euclidean distance  $d_{NE}(A, B)$ :

$$d_{NE}(A, B) = \sqrt{\frac{1}{2} \sum_{x \in X} [(\mu_A(x) - \mu_B(x))^2 + (\nu_A(x) - \nu_B(x))^2]}$$

Wang and Xin proposed approach

$$d_{WX}(A, B) = \frac{1}{2} \sum_{x \in X} \left[ \frac{|\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)|}{4} + \frac{\max\{|\mu_A(x) - \mu_B(x)|, |\nu_A(x) - \nu_B(x)|\}}{2} \right]$$

Szmidt and Kacprzyk further modified these distances for IFSs by considering the three parameters of IFS (Szmidt et al., 2000, Szmidt et al., 2004): degree of membership  $\mu$ , degree of non-membership  $\nu$  and degree of hesitancy  $\pi$  given by:

$$D_1 = \sum_{i=1}^n \frac{1}{2} [|0-0| + |0-0| + |0-0|]$$

$$D_2 = \sum_{i=1}^n \frac{1}{2} [|0-0| + |0-0| + |0-0|]$$

$$D_3 = \sqrt{\frac{1}{2} \sum_{i=1}^n [(x_i - y_i)^2 + (y_i - z_i)^2 + (z_i - x_i)^2]}$$

$$D_4 = \sqrt{\frac{1}{2} \sum_{i=1}^n [(x_i - y_i)^2 + (y_i - z_i)^2 + (z_i - x_i)^2]}$$

**Grzegorzewski's distance measure**

$$D_5 = \sum_{i=1}^n \max\{|0-0|, |0-0|\}$$

**Yang and Francisco's distance measure**

$$D_6 = \sum_{i=1}^n \max\{|0-0|, |0-0|, |0-0|\}$$

**Jin Han Park's distance measure**

$$D_7 = \frac{1}{4} \sum_{i=1}^n (|x_i - y_i| + |y_i - z_i| + |z_i - x_i| + \max\{|0-0|, |0-0|, |0-0|\})$$

**Song's distance measure**

$$D_8 = 1 - \sum_{i=1}^n \frac{1}{2} \left( \frac{\sqrt{(x_i - y_i)^2 + 2\sqrt{(x_i - y_i)(y_i - z_i)} + \sqrt{(x_i - y_i)(y_i - z_i)} + \sqrt{(1 - (x_i - y_i))(1 - (y_i - z_i))}}{\sqrt{(1 - (x_i - y_i))(1 - (y_i - z_i))}} \right)$$

**Hatzimichailidis's distance measure**

$$D_9 = \frac{\|x - y\|_2 + \|x - z\|_2}{2}, \text{ where } \alpha = \frac{1}{2}$$

$$D_{10} = \frac{\|x - y\|_2 + \|x - z\|_2}{2}, \text{ where } \alpha = 1 - \alpha$$

$$D_{11} = \frac{\|x - y\|_2 + \|x - z\|_2}{2}, \text{ where } \alpha = \min(1, 1 - \alpha)$$

$$d(A, B) = \frac{\| \mu_A(x) - \mu_B(x) \| + \| \nu_A(x) - \nu_B(x) \|}{2},$$

where  $\mu(x) = \max(1 - \nu, \dots)$

$$d(A, B) = \frac{\| \mu_A(x) - \mu_B(x) \| + \| \nu_A(x) - \nu_B(x) \|}{2},$$

where  $\mu(x) = \max(\dots)$

$$d(A, B) = \frac{\| \mu_A(x) - \mu_B(x) \| + \| \nu_A(x) - \nu_B(x) \|}{2},$$

where  $\mu(x) = \max(\dots)$

$$d(A, B) = \frac{\| \mu_A(x) - \mu_B(x) \| + \| \nu_A(x) - \nu_B(x) \|}{2},$$

where  $\mu(x) = 1$  for  $x \in X$ , for  $x \notin X$

Minxia Luo and Ruirui Zhao's distance measure

$$d(A, B) = \frac{\| \mu_A(x) - \mu_B(x) \| + \| \nu_A(x) - \nu_B(x) \|}{2} + \frac{\| \pi_A(x) - \pi_B(x) \|}{2},$$

$$\| \mu_A - \mu_B \| = \sqrt{\max(\dots)}$$

is the largest non negative eigenvalue of positive definite Hermitian matrix  $\Pi \Pi^T$

**PROPOSE DISTANCE MEASURE BETWEEN IFSS**

In this section we propose new distance measure for IFSSs defined on universe of discourse X by taking into account all the three parameters: the degree of membership  $\mu(x)$ , the degree of non-membership  $\nu(x)$  and concept of hesitancy degree  $\pi(x)$ , we define the distance measure on IFSSs as follows. For any two IFSSs

$$A = \{ (x, \mu_A(x), \nu_A(x), \pi_A(x)) \mid x \in X \}$$

$B = \{ (x, \mu_B(x), \nu_B(x), \pi_B(x)) \mid x \in X \}$  defined on universe of discourse X. Then we have

$$d(A, B) = \frac{1}{2} \left[ \sum_{x \in X} \left( \frac{\max\{ |\mu_A(x) - \mu_B(x)|, |\nu_A(x) - \nu_B(x)| \}}{2} + \frac{|\pi_A(x) - \pi_B(x)|}{2} \right) \right]$$

**Theorem**

$d(A, B)$  is the degree of distance between two IFSSs A and B in  $\{0, 1, 2, \dots, \dots\}$ .

**Proof**

Let  $A = \{ (x, \mu_A(x), \nu_A(x), \pi_A(x)) \mid x \in X \}$  and  $B = \{ (x, \mu_B(x), \nu_B(x), \pi_B(x)) \mid x \in X \}$  be two IFSSs. Obviously,

$\mu(x)$  satisfies the axioms 1. of definition 2.3. 2. If  $\mu_A(x) = \mu_B(x)$  and  $\nu_A(x) = \nu_B(x)$  then  $\pi_A(x) = \pi_B(x)$

Conversely, if  $d(A, B) = 0$  Then  $A=B$ . Therefore,  $d(A, B) = 0 \iff A=B$

D3. Clearly,  $d(A, A) = 0$

D4. For any IFSSs  $A = \{ (x, \mu_A(x), \nu_A(x), \pi_A(x)) \mid x \in X \}$  and  $B = \{ (x, \mu_B(x), \nu_B(x), \pi_B(x)) \mid x \in X \}$  then we have,  $0 \leq d(A, B) \leq 1$  and  $0 \leq d(B, A) \leq 1$ .....(i) Thus,

$$d(A, B) = \frac{1}{2} \sum_{x \in X} \left[ \frac{\max\{ |\mu_A(x) - \mu_B(x)|, |\nu_A(x) - \nu_B(x)| \}}{2} + \frac{|\pi_A(x) - \pi_B(x)|}{2} \right] \dots(ii)$$

$$d(B, A) = \frac{1}{2} \sum_{x \in X} \left[ \frac{\max\{ |\mu_B(x) - \mu_A(x)|, |\nu_B(x) - \nu_A(x)| \}}{2} + \frac{|\pi_B(x) - \pi_A(x)|}{2} \right] \dots(iii)$$

$$\frac{1}{d(A, B)} = \frac{1}{2} \left[ \sum_{x \in X} \left( \frac{\max\{ |\mu_A(x) - \mu_B(x)|, |\nu_A(x) - \nu_B(x)| \}}{2} + \frac{|\pi_A(x) - \pi_B(x)|}{2} \right) \right] \dots(iv)$$

From (i), it is clear that

$$\begin{aligned} &|\mu_A(x) - \mu_B(x)| \leq |\mu_B(x) - \mu_A(x)| \\ &\& |\nu_A(x) - \nu_B(x)| \leq |\nu_B(x) - \nu_A(x)| \\ &|\pi_A(x) - \pi_B(x)| \leq |\pi_B(x) - \pi_A(x)| \\ &\& |\pi_A(x) - \pi_B(x)| \leq |\pi_B(x) - \pi_A(x)| \end{aligned}$$



obtained from the expert committee. The weights can be expressed in the matrix format as follows:

$$\begin{matrix}
 & 1 & 2 & \dots & \dots & \dots \\
 1 & \left[ \begin{array}{cccc}
 \tilde{\pi}_{11} & \tilde{\pi}_{12} & \dots & \dots \\
 \tilde{\pi}_{21} & \tilde{\pi}_{22} & \dots & \dots \\
 \vdots & \vdots & \dots & \dots \\
 \tilde{\pi}_{12} & \tilde{\pi}_{22} & \dots & \dots
 \end{array} \right]
 \end{matrix}$$

**Step3**

Calculate the distances between the ratings of the alternatives and the weights of the relevant criterion.

The relation between the alternatives and the different groups can be established in the matrix format as follows:

$$\begin{matrix}
 & 1 & 2 & \dots & \dots & \dots \\
 1 & \left[ \begin{array}{cccc}
 d_{11} & d_{12} & \dots & \dots \\
 d_{21} & d_{22} & \dots & \dots \\
 \vdots & \vdots & \dots & \dots \\
 d_{12} & d_{22} & \dots & \dots
 \end{array} \right]
 \end{matrix}$$

where,  $d_{ij}$  is distance of the alternative  $i$  from the weights of the criteria  $j$  for belonging to a certain group.

**Step4**

**Table 2.** Patients-symptoms intuitionistic fuzzy relation.

P→S	Temperature	Headache	Stomach pain	Cough	Chest pain
Sankar	(0.8,0.1)	(0.6,0.1)	(0.2,0.8)	(0.6,0.1)	(0.1,0.6)
Abhijit	(0.0,0.8)	(0.4,0.4)	(0.6,0.1)	(0.1,0.7)	(0.1,0.8)
Amlan	(0.8,0.1)	(0.8,0.1)	(0.0,0.6)	(0.2,0.7)	(0.0,0.5)
Aburba	(0.6,0.1)	(0.5,0.4)	(0.3,0.4)	(0.7,0.2)	(0.3,0.4)

**Table 3.** symptom-disease intuitionistic fuzzy relation.

S→D	Viral Fever	Malaria	Typhoid	Stomach	Chest Problem
Temperature	(0.4,0.0)	(0.7,0.0)	(0.3,0.3)	(0.1,0.7)	(0.1,0.8)
Headache	(0.3,0.5)	(0.2,0.6)	(0.6,0.1)	(0.2,0.4)	(0.0,0.8)
Stomachpain	(0.1,0.7)	(0.0,0.9)	(0.2,0.7)	(0.8,0.0)	(0.2,0.8)
Cough	(0.4,0.3)	(0.7,0.0)	(0.2,0.6)	(0.2,0.7)	(0.2,0.8)
Chestpain	(0.1,0.7)	(0.1,0.8)	(0.1,0.9)	(0.2,0.7)	(0.8,0.1)

Now, evaluating the distance value using the proposed distance measure for the above data sets (Table 2 & Table 3) the following Table 4 is constructed:

**Table 4.** Distance value and patient-disease intuitionistic fuzzy relation.

If the distance is less, it indicates that the alternative is more nearer to its relevant group. Therefore, rank the alternative according to the decreasing order of their distances.

**CASE STUDY**

**Case Study 1**

In this section, two hypothetical case studies have been carried out to perform medical diagnosis using the concept of IFSs based on the proposed distance measure. Here, it is proposed to take into account the three parameters characterization of IFSs: the membership degree, the non-membership degree and decision maker’s hesitancy degree.

Let  $P = \{Sankar, Abhijit, Amlan, Apurba\}$  be the set of patients,  $S = \{\text{temperature, headache, stomach pain, cough, chest pain}\}$  be the set of symptoms,  $D = \{\text{viral fever, Malaria, typhoid, stomach problem, chest problem}\}$  be the set of diseases. Our intention is to carry out the right decision for each patient  $i = 1, 2, 3, 4$  from the set of symptoms  $j = 1, 2, 3, 4, 5$  for each disease  $k = 1, 2, 3, 4, 5$ .

The patient-symptom intuitionistic fuzzy relation  $P \rightarrow S$  and symptom-disease intuitionistic fuzzy relation  $S \rightarrow D$  are given in Table 2 and Table 3 respectively.

P→D	Viral Fever	Malaria	Typhoid	Stomach	Chest Problem
Sankar	0.1770	0.1473	0.1869	0.3816	0.4168
Abhijit	0.2725	0.3705	0.2078	0.0812	0.2985
Amlan	0.2500	0.3046	0.2089	0.3530	0.4155
Apurba	0.1716	0.1891	0.2441	0.2966	0.3785

**Case Study 2**

Let there be four Patients  $P=\{\text{Shyam, Soumendra, Jadov, Joydeep}\}$  and the set of symptoms  $S = \{\text{Headache, Acidity, Burning Eyes, Back pain, Depression}\}$  Let the set of Disease be  $D=\{\text{Stress, Ulcer, Vision problem, Spinal problems, Blood pressure}\}$ . The

patients-symptoms intuitionistic fuzzy relation  $P \rightarrow S$  and the symptoms-disease intuitionistic fuzzy relation  $S \rightarrow D$  are shown in the Table 5 and Table 6 respectively. The patients-disease relation  $P \rightarrow D$  is constructed in Table 7.

**Table 5.** Patients-symptoms intuitionistic fuzzy relation.

P→S	Headache	Acidity	Burning Eyes	Back Pain	Depression
Shyam	(0.9, 0.1)	(0.7, 0.2)	(0.2, 0.8)	(0.7, 0.2)	(0.2, 0.7)
Soumendra	(0.0, 0.7)	(0.4, 0.5)	(0.6, 0.2)	(0.2, 0.7)	(0.1, 0.2)
Jadov	(0.7, 0.1)	(0.7, 0.1)	(0.0, 0.5)	(0.1, 0.7)	(0.0, 0.6)
Joydeep	(0.5, 0.1)	(0.4, 0.3)	(0.4, 0.5)	(0.8, 0.2)	(0.3, 0.4)

**Table 6.** Symptom-disease intuitionistic fuzzy relation.

S→D	Stress	Ulcer	Vision Problem	Spinal Problem	Blood Pressure
Headache	(0.3, 0.0)	(0.0, 0.6)	(0.2, 0.2)	(0.2, 0.8)	(0.2, 0.8)
Acidity	(0.3, 0.5)	(0.2, 0.6)	(0.5, 0.2)	(0.1, 0.5)	(0.0, 0.7)
Burning Eyes	(0.2, 0.8)	(0.0, 0.8)	(0.1, 0.7)	(0.7, 0.0)	(0.2, 0.8)
Back Pain	(0.7, 0.3)	(0.5, 0.0)	(0.2, 0.6)	(0.1, 0.7)	(0.1, 0.8)
Depression	(0.2, 0.6)	(0.1, 0.8)	(0.2, 0.8)	(0.2, 0.7)	(0.8, 0.1)

P→D	Stress	Ulcer	Vision Problem	Spinal Problem	Blood Pressure
Shyam	0.15534	0.2647	0.2115	0.393	0.404
Soumendra	0.2939	0.2777	0.2496	0.1475	0.2521
Jadov	0.2557	0.3482	0.1902	0.3451	0.3846
Joydeep	0.1229	0.2507	0.208	0.3186	0.3558

**Table 7.** distance value and patients-disease intuitionistic fuzzy relation.

**RESULTS**

The principle of minimum distance degree states that the lower distance degree of alternative signifies a proper diagnosis. The obtained results have been highlighted by bold character in Table 4 and in Table 7. From the Table 4 the proper diagnosis are Sankar suffers from Malaria, Ahbijit suffer from Stomach pain, Amlan suffers from Typhoid and Apurba suffers from

Viral fever, which coincides with the results (Pramanik et al., 2015, Szmidi et al., 2001, Wei et al., 2011, Luo et al., 2018) and from the Table 7, Shyam suffers from stress, Soumendra suffers from spinal problems, Jadov suffers from vision problem and Joydeep suffers from stress. From numerical comparison, it has been observed that the proposed distance measure can overcome the counterintuitive cases whereas most of the existing distance measure failed to reflect the same. Thus, the obtained results in the above medical diagnosis problems by using the proposed distance

measure can be recognised as more confident and improved results.

## DISCUSSION

Various studies have been done so far in medical diagnosis using fuzzy set, interval valued fuzzy set, intuitionistic fuzzy, interval valued intuitionistic fuzzy set etc. It is seen that IFS gained more popularity in medical diagnosis because of its interesting concept of membership and non-membership degree. Different distance measures are developed and have applied in many different areas under IFSs environment but many of them fails to handle some critical situation (Numerical comparison). Therefore, the use of such distance measures may mislead some illogical results or may not evaluate the results properly. To obtain proper and logical results in various problems in different branches, the proposed distance measure can be implemented as a novel and efficient distance measure.

## CONCLUSION

This paper attempts to devise a novel distance measure between IFSs to effectively resolve the shortcomings of the existing distance measures. The proposed distance measure considers all the three assignment, the hesitancy degree  $\pi$ , the membership degree  $\mu$  and non-membership degree  $\nu$  instead of taking only the membership degree  $\mu$  and non-membership degree  $\nu$ . The validity and efficiency of the proposed method has been shown by studying a comparative analysis through numerical illustrations and medical decision making analysis via the proposed approach. In the future work, the presented approach has been extended to Pythagorean fuzzy sets and some other uncertain environment (Garg et al., 2018, Garg, 2018, Song et al., 2017).

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